Bayesian statistics with R
4. A detour with priors

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A detour to explore priors

Influence of the prior

Prior $Beta(0.5, 0.5)$ and posterior survival $Beta(19.5, 38.5)$
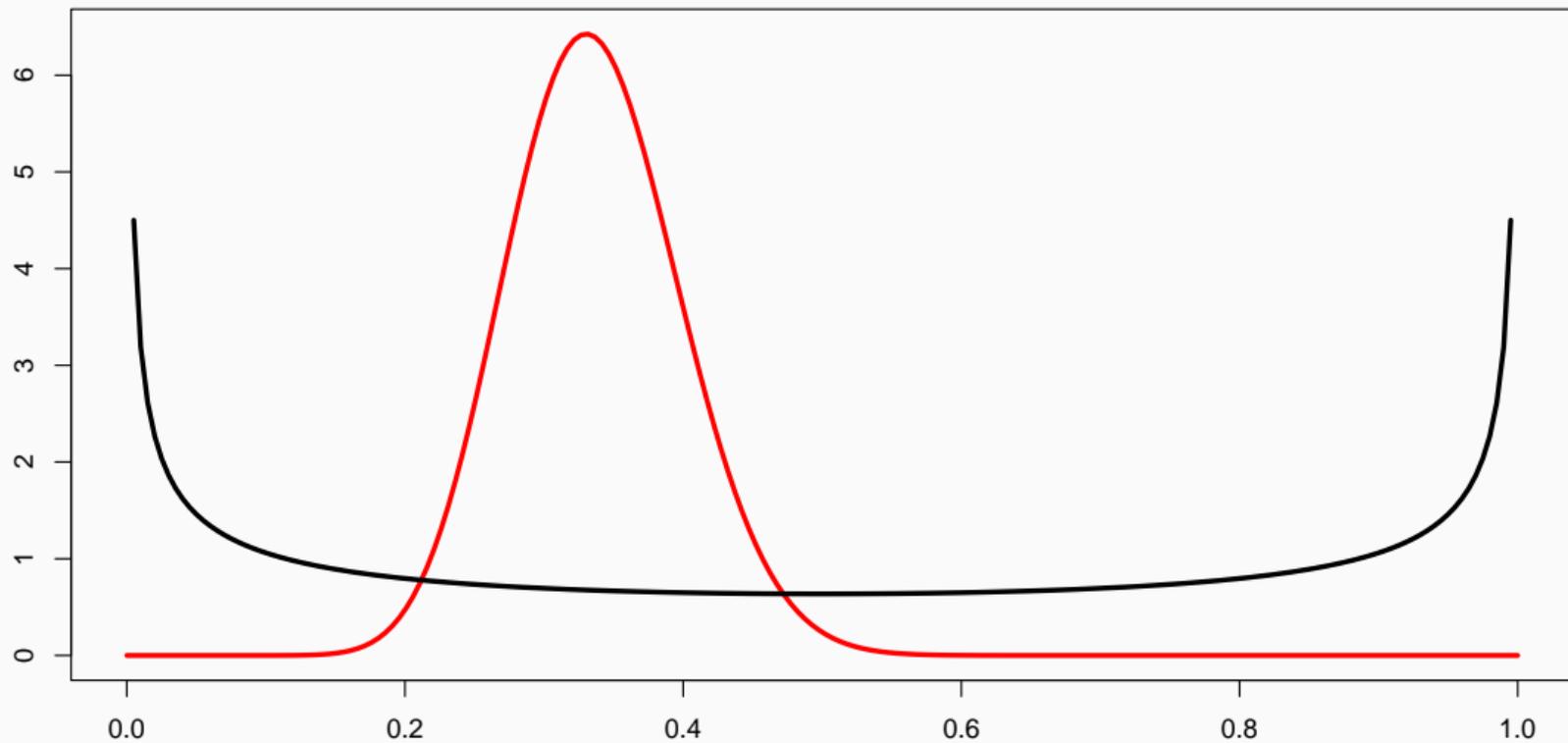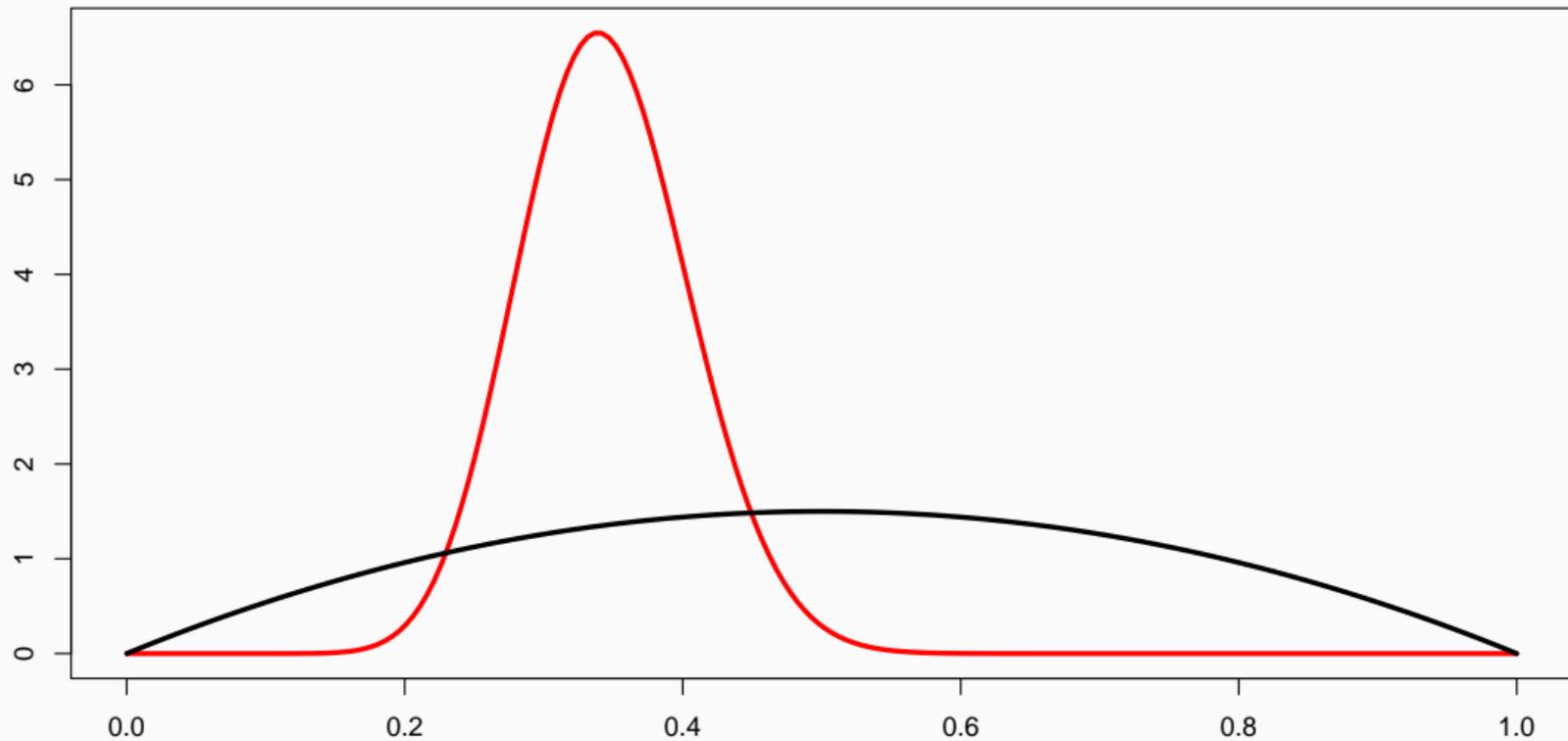
Prior *Beta*(20, 1) **and posterior survival** *Beta*(39, 49)
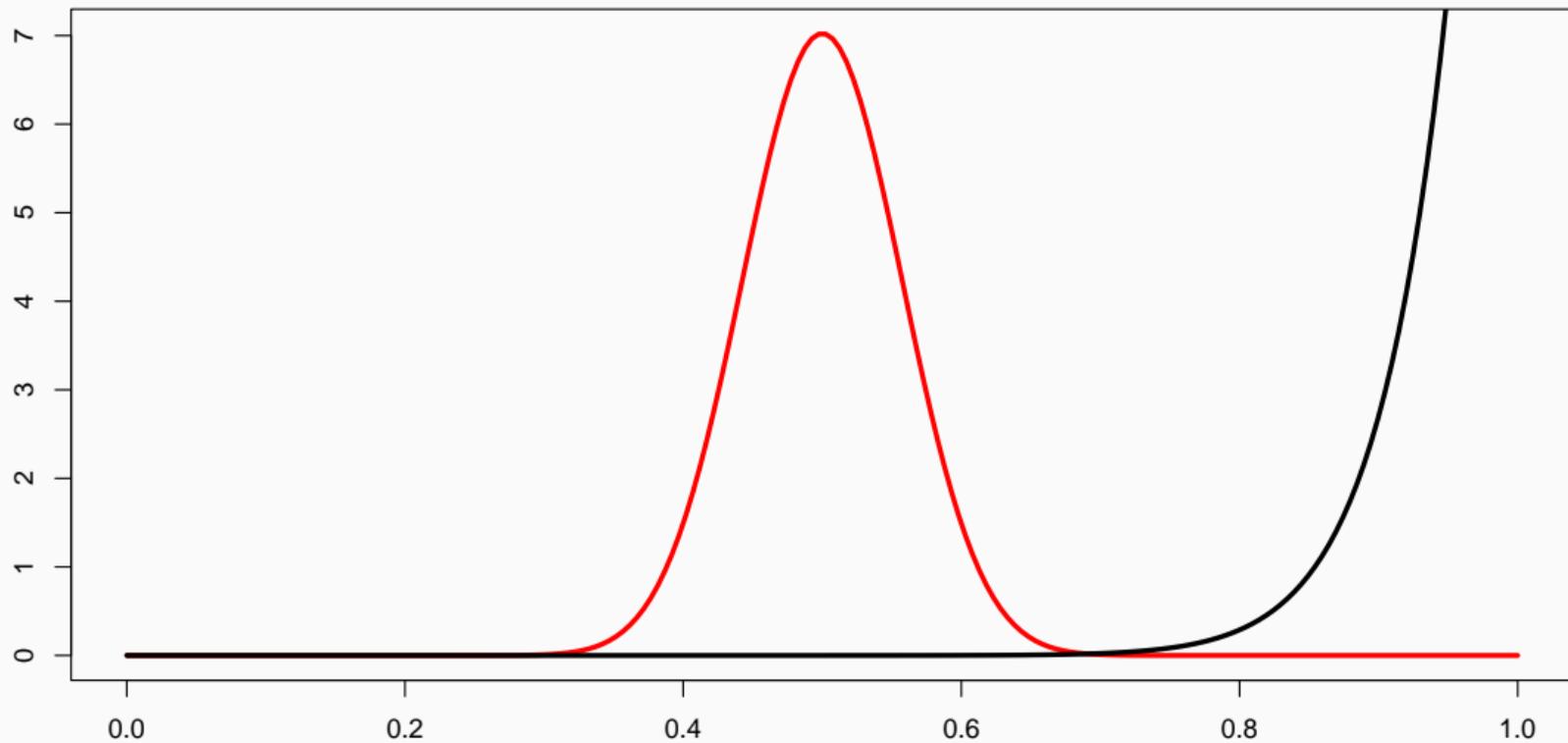
The role of the prior

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- We assume a vague prior:

$$\phi_{prior} \sim \text{Beta}(1, 1) = \text{Uniform}(0, 1)$$

Notation

- $y_{i,t} = 1$ if individual i detected at occasion t and 0 otherwise
- $z_{i,t} = 1$ if individual i alive between occasions t and $t+1$ and 0 otherwise

$$y_{i,t} \mid z_{i,t} \sim \text{Bernoulli}(p\, z_{i,t}) \qquad \text{[likelihood (observation eq.)]}$$
$$z_{i,t+1} \mid z_{i,t} \sim \text{Bernoulli}(\phi\, z_{i,t}) \qquad \text{[likelihood (state eq.)]}$$
$$\phi \sim \text{Beta}(1,1) \qquad \text{[prior for } \phi\text{]}$$
$$p \sim \text{Beta}(1,1) \qquad \text{[prior for p]}$$

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- Width of credible interval is 0.47 (vague prior) vs. 0.30 (informative prior).

- Huge increase of precision in posterior inference (40% gain)!
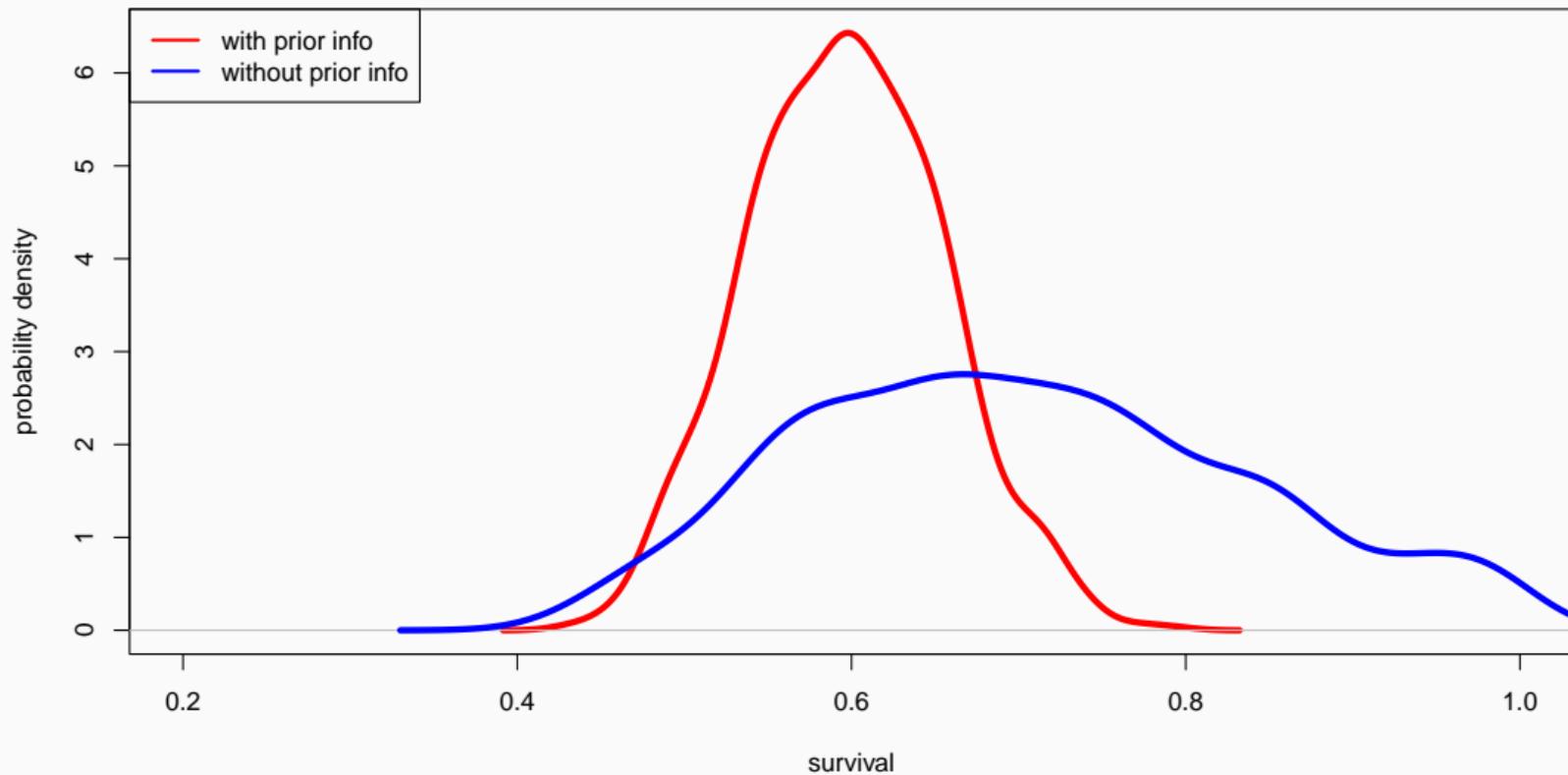
Prior elicitation via moment matching

Remember the Beta distribution

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- If $X \sim Beta(\alpha, \beta)$, then the first and second moments of X are:

$$\mu = \mathsf{E}(X) = \frac{\alpha}{\alpha + \beta}$$

$$\sigma^2 = \mathsf{Var}(X) = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)}$$

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- Now we look for values of α and β that match the observed moments of the Beta distribution (μ and σ^2).

- We need another set of equations:

$$\alpha = \left(\frac{1-\mu}{\sigma^2} - \frac{1}{\mu} \right) \mu^2$$

$$\beta = \alpha \left(\frac{1}{\mu} - 1 \right)$$

- For our model, that means:

```
(alpha <- ( (1 - 0.57)/(0.073*0.073) - (1/0.57) )*0.57^2)
#> [1] 25.64636
(beta <- alpha * ( (1/0.57) - 1))
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- Now use $\phi_{prior} \sim \text{Beta}(\alpha = 25.6, \beta = 19.3)$ instead of
  $\phi_{prior} \sim \text{Normal}(0.57, 0.073^2)$

Your turn: Practical 3

Prior predictive checks

Linear regression

Unreasonable prior $\beta \sim N(0, 1000^2)$
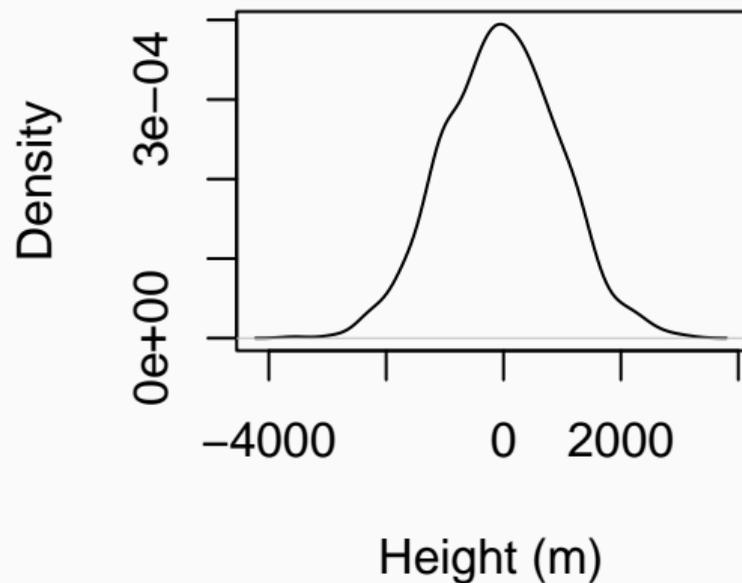
Reasonable prior $\beta \sim N(2, 0.5^2)$
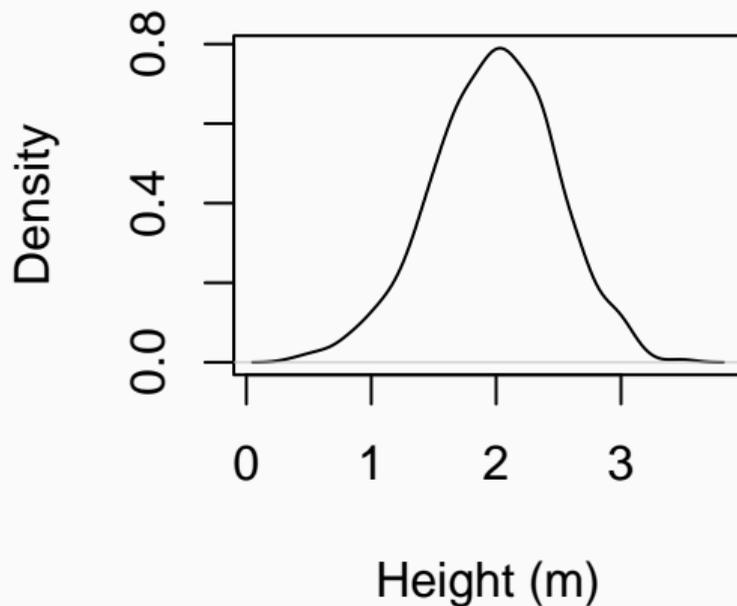
Logistic regression

Unreasonable $\text{logit}(\phi) = \beta \sim N(0, 10^2)$
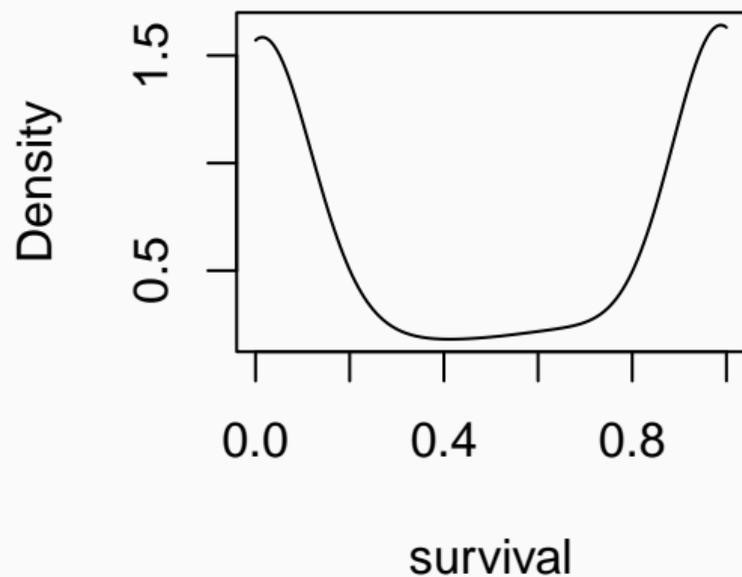
Reasonable $\text{logit}(\phi) = \beta \sim N(0, 1.5^2)$

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