Bayesian statistics with R
1. An introduction to Bayesian inference

Olivier Gimenez

November-December 2024

Credit where credit's due

- Ruth King, Byron Morgan, Steve Brooks (our workshops and Bayesian analysis for population ecology book).
- Richard McElreath (Statistical rethinking book and lecture videos).
- Jim Albert and Jingchen Hu (Probability and Bayesian modelling book).
- Materials shared by Tristan Marh, Jason Matthiopoulos, Francisco Rodriguez Sanchez, Kerrie Mengersen and Mark Lai.

Slides, code and data

- All material prepared with `R`.
- `R Markdown` used to write reproducible material.
- Dedicated website https://oliviergimenez.github.io/bayesian-stats-with-R/.

Objectives

- Try and demystify Bayesian statistics, and what we call MCMC.
- Make the difference between Bayesian and Frequentist analyses.
- Understand the Methods section of ecological papers doing Bayesian stuff.
- Run Bayesian analyses, safely hopefully.

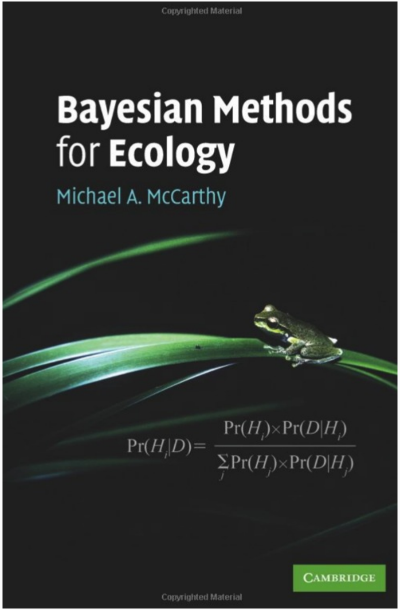

BRACE YOURSELF

What is on our plate?

1. An introduction to Bayesian inference
2. The likelihood
3. Bayesian analyses by hand
4. A detour to explore priors
5. Markov chains Monte Carlo methods (MCMC)
6. Bayesian analyses in `R` with the `Jags` software
7. Contrast scientific hypotheses with model selection
8. Heterogeneity and multilevel models (aka mixed models)

I want moooooore


I ONLY LIKE TWO THINGS:
THEY'RE BOTH BOOKS


Copyrighted Material
Bayesian Methods for Ecology
Michael A. McCarthy
$\Pr(H_i|D) = \frac{\Pr(H_i) \times \Pr(D|H_i)}{\sum_j \Pr(H_j) \times \Pr(D|H_j)}$
CAMBRIDGE
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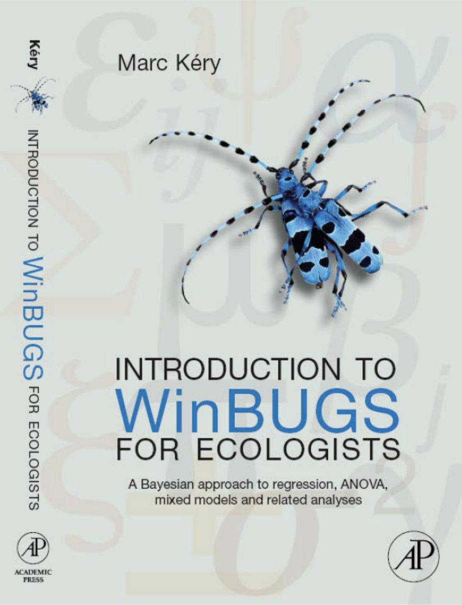
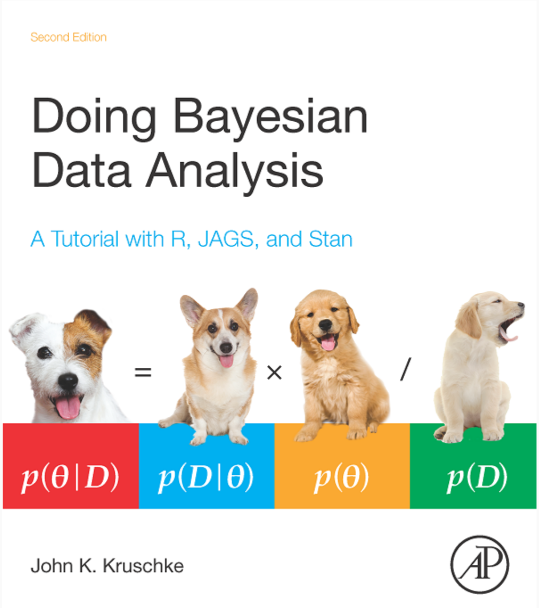
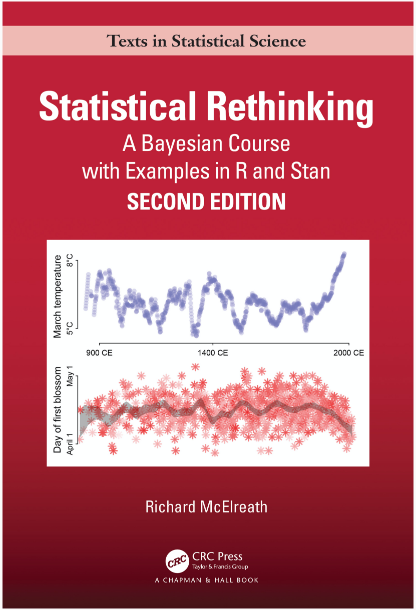
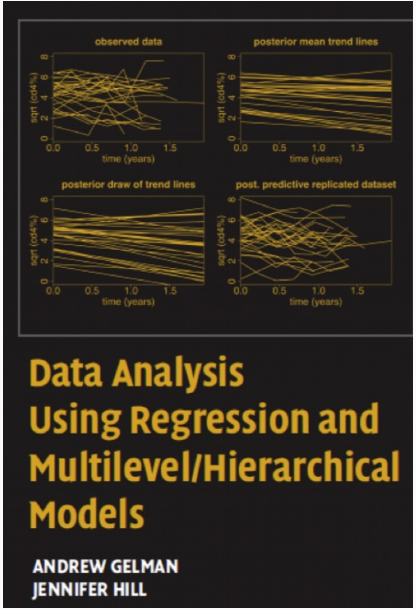
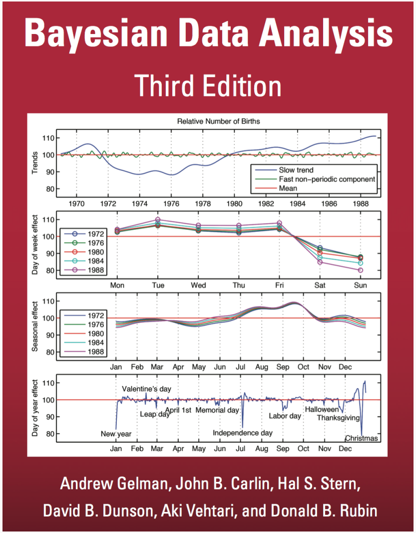


Kéry
INTRODUCTION TO WinBUGS FOR ECOLOGISTS
ACADEMIC PRESS
Marc Kéry
INTRODUCTION TO
WinBUGS
FOR ECOLOGISTS
A Bayesian approach to regression, ANOVA,
mixed models and related analyses


Second Edition
Doing Bayesian Data Analysis
A Tutorial with R, JAGS, and Stan
$p(\theta|D)$ = $p(D|\theta)$ × $p(\theta)$ / $p(D)$
John K. Kruschke

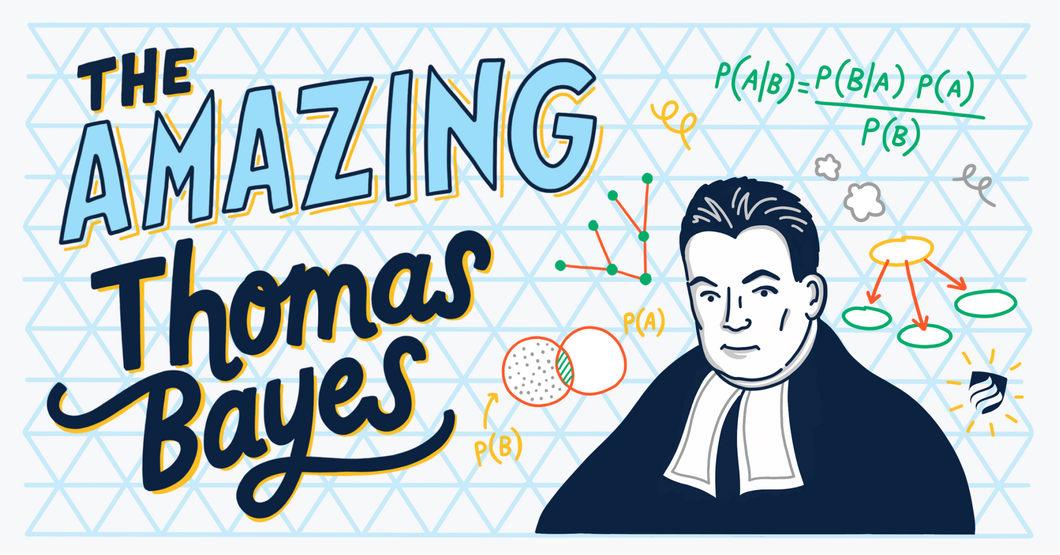

Texts in Statistical Science
Statistical Rethinking
A Bayesian Course
with Examples in R and Stan
SECOND EDITION
March temperature
8°C
5°C
900 CE
1400 CE
2000 CE
Day of first blossom
May 1
April 1
Richard McElreath
CRC Press
Taylor & Francis Group
A CHAPMAN & HALL BOOK


observed data
posterior mean trend lines
posterior draw of trend lines
post. predictive replicated dataset
sqrt (cd4%)
time (years)
Data Analysis
Using Regression and
Multilevel/Hierarchical
Models
ANDREW GELMAN
JENNIFER HILL



Free at http://www.stat.columbia.edu/~gelman/book/

What is Bayesian inference?


THE AMAZING
Thomas Bayes
P(A|B)= P(B|A) P(A) / P(B)
P(A)
P(B)

A reminder on conditional probabilities

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HOW TO CURE VAMPIRES?
imgflip.com

Screening for vampirism

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- The chance of a negative test given you are mortal is $\Pr(-|\text{mortal}) = 0.95$ (**specificity**).

What is the question?

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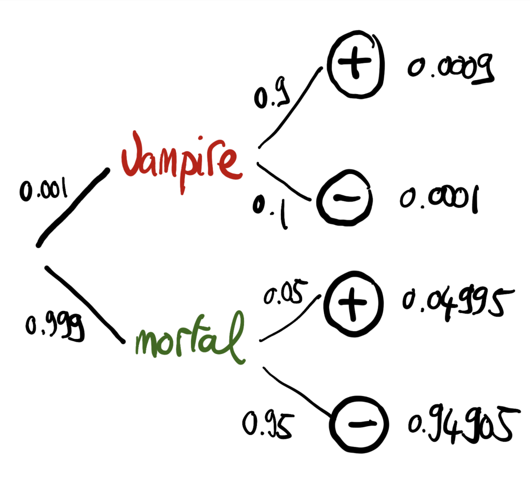
- From the perspective of a person: Given that the test is positive, what is the probability that this person is a vampire? $\Pr(\text{vampire}|+) = ?$
- Assume that vampires are rare, and represent only 0.1% of the population. This means that $\Pr(\text{vampire}) = 0.001$.

What is the answer? Bayes' theorem to the rescue!



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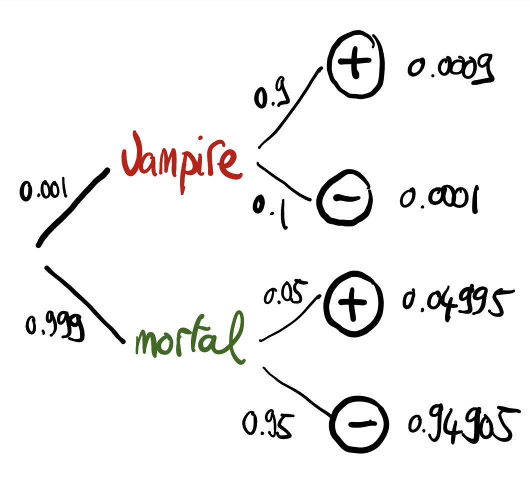

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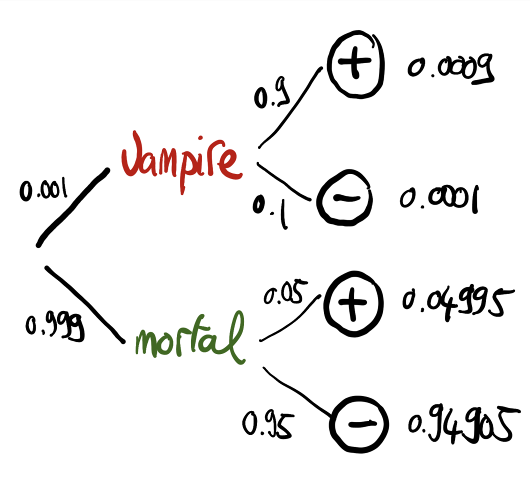

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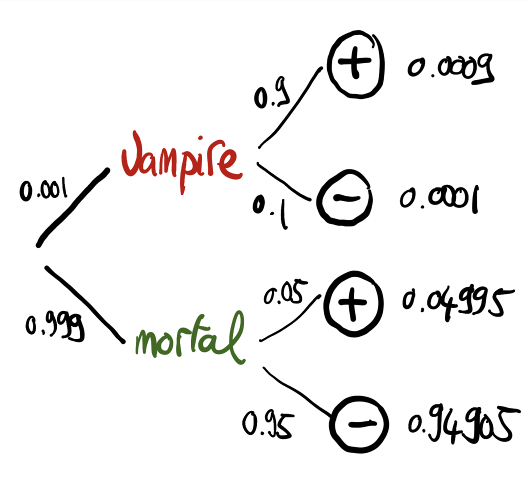

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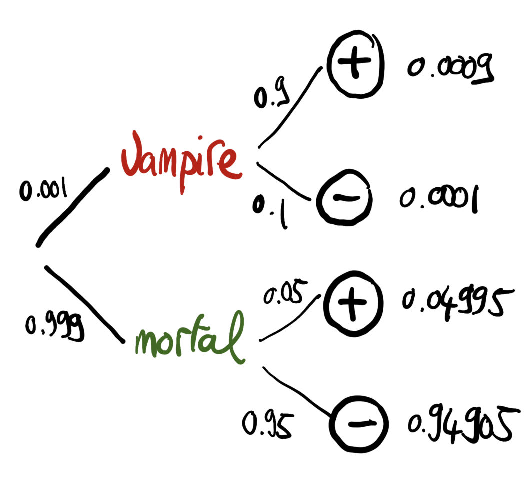

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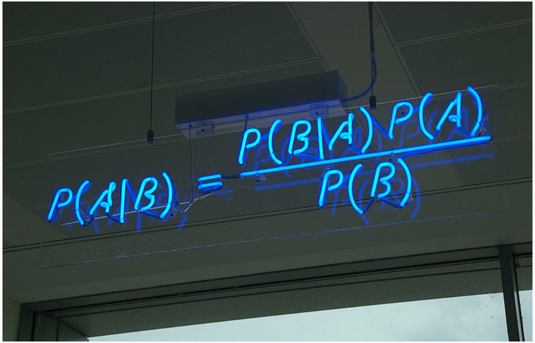
$$\Pr(\text{vampire}|+) = \frac{\Pr(+|\text{vampire})\ \Pr(\text{vampire})}{\Pr(+)}$$

Your turn: Practical 1

Bayes' theorem

- A theorem about conditional probabilities.
- $\Pr(B \mid A) = \dfrac{\Pr(A \mid B)\ \Pr(B)}{\Pr(A)}$



Bayes' theorem

- Easy to mess up with letters. Might be easier to remember when written like this:

$$\Pr(\text{hypothesis} \mid \text{data}) = \frac{\Pr(\text{data} \mid \text{hypothesis}) \; \Pr(\text{hypothesis})}{\Pr(\text{data})}$$

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- The "hypothesis" is typically something unobserved or unknown. It's what you want to learn about using the data.
- For regression models, the "hypothesis" is a parameter (intercept, slopes or error terms).
- Bayes theorem tells you the probability of the hypothesis given the data.

What is doing science after all?

How plausible is some hypothesis given the data?

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Why is Bayesian statistics not the default?

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- Due to practical problems of implementing the Bayesian approach, and some wars of male statisticians's egos, little advance was made for over two centuries.
- Recent advances in computational power coupled with the development of new methodology have led to a great increase in the application of Bayesian methods within the last two decades.

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- The frequentist approach (**maximum likelihood estimation** – MLE) assumes that the parameters are fixed, but have unknown values to be estimated.
- Classical estimates generally provide a point estimate of the parameter of interest.
- The Bayesian approach assumes that the parameters are not fixed but have some fixed unknown distribution - a distribution for the parameter.

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- And then updates these beliefs on the basis of observed data.
- This updating procedure is based upon the Bayes' Theorem:

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- Translates into:

$$\Pr(\theta \mid \text{data}) = \frac{\Pr(\text{data} \mid \theta)\ \Pr(\theta)}{\Pr(\text{data})}$$

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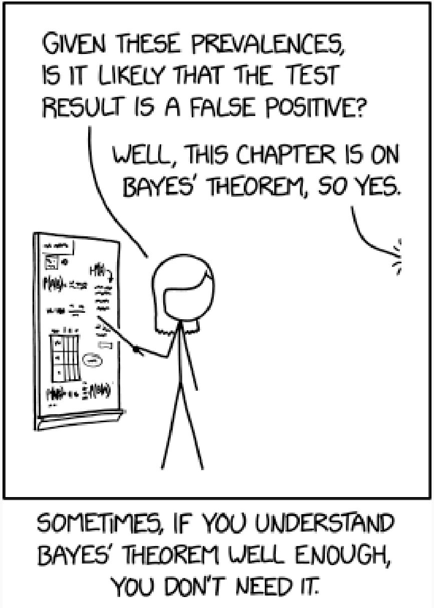
- $\Pr(\text{data}) = \int \Pr(\text{data} \mid \theta)\ \Pr(\theta) d\theta$: Possibly high-dimensional integral, difficult if not impossible to calculate. This is one of the reasons why we need simulation (MCMC) methods - more soon.


GIVEN THESE PREVALENCES, IS IT LIKELY THAT THE TEST RESULT IS A FALSE POSITIVE?
WELL, THIS CHAPTER IS ON BAYES' THEOREM, SO YES.
SOMETIMES, IF YOU UNDERSTAND BAYES' THEOREM WELL ENOUGH, YOU DON'T NEED IT.